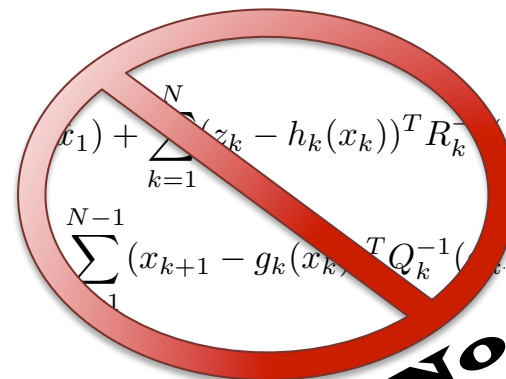


# A Conceptual Introduction to Geophysical Inversion



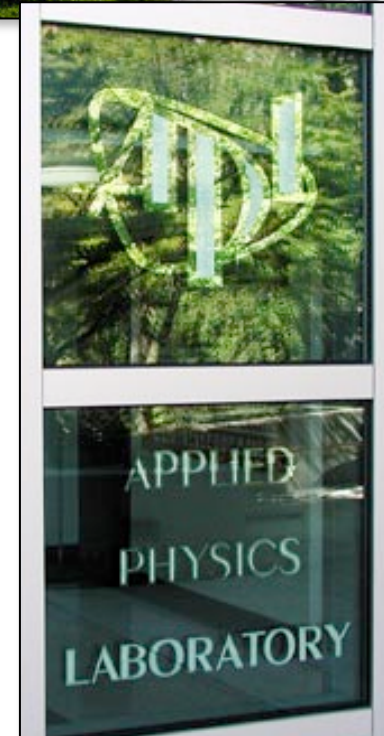
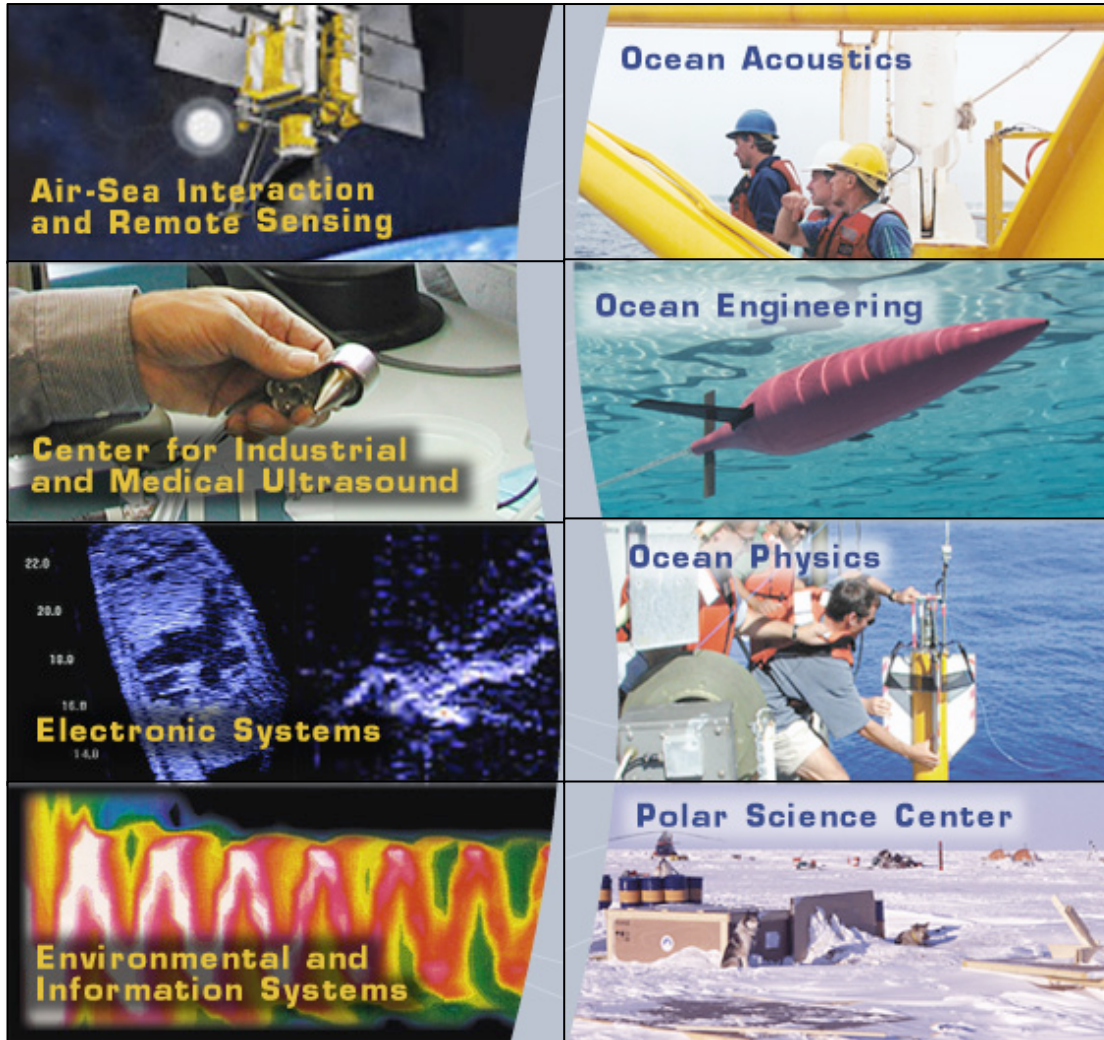
Andy Ganse  
ESS & Applied Physics Lab  
University of Washington  
12 Mar 2012



**No Math  
in this one!**

# A little about APL

*(my stomping ground...)*



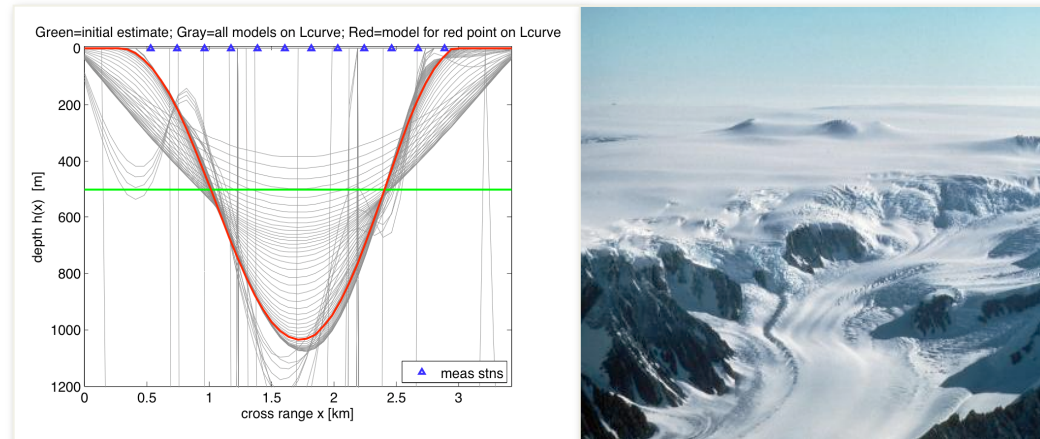
# A conceptual / non-technical talk

*A terminology extravaganza in a sequence of dichotomies...*

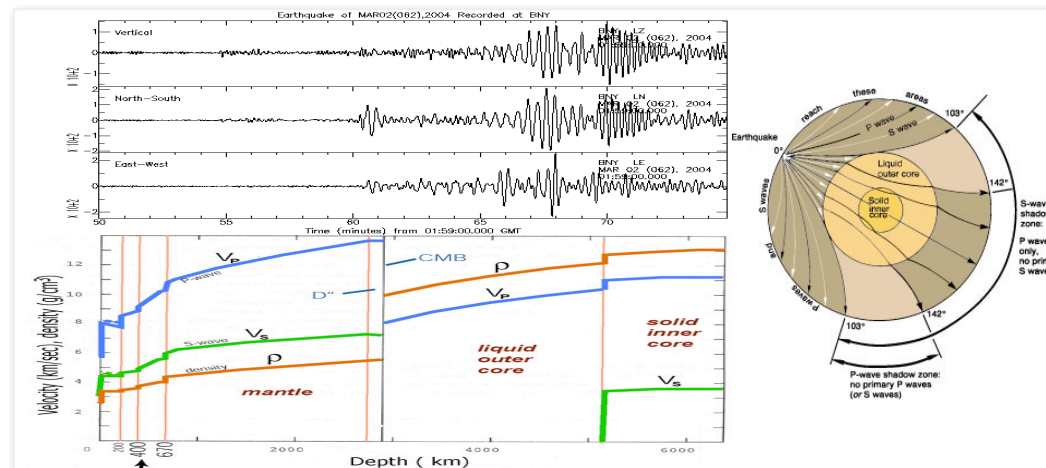
- Intro via examples
- Data vs. Model
- Deduction vs. Induction
- Probability vs. statistics
- Frequentist vs. Bayesian
- Math vs. Earth Science
- Parameter estimation vs. inversion
- Uncertainty vs. Resolution
- Linear vs. Nonlinear
- Recommended reading

# Intro via examples

*Glacier gravimetry: estimate glacier cross-section from gravity measurements*



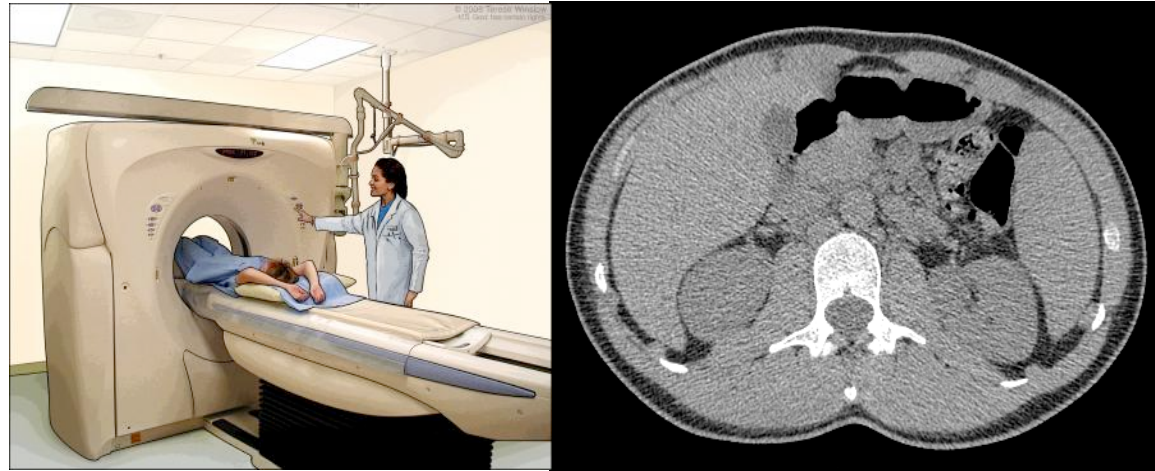
*Global seismic inversion: estimate Earth's interior wavespeeds & densities from EQ seismograms*



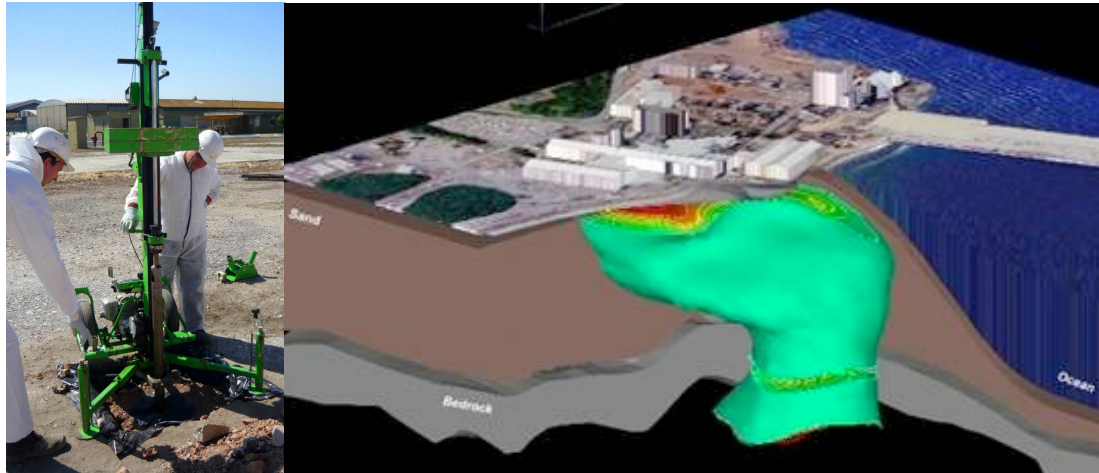


# Intro via examples

*Computerized Tomography (CT) scans: estimate 3D body interior densities from Xray atten*

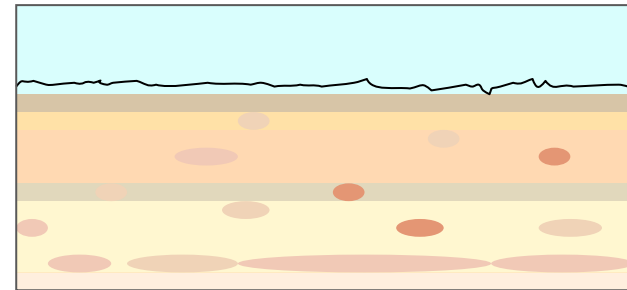
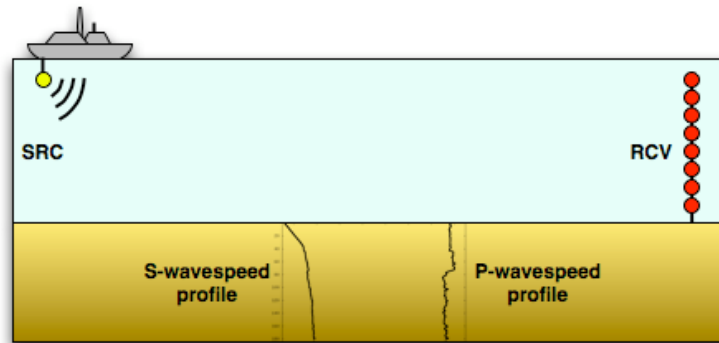


*Groundwater contamination: estimate source leakage function from groundwater samplings*

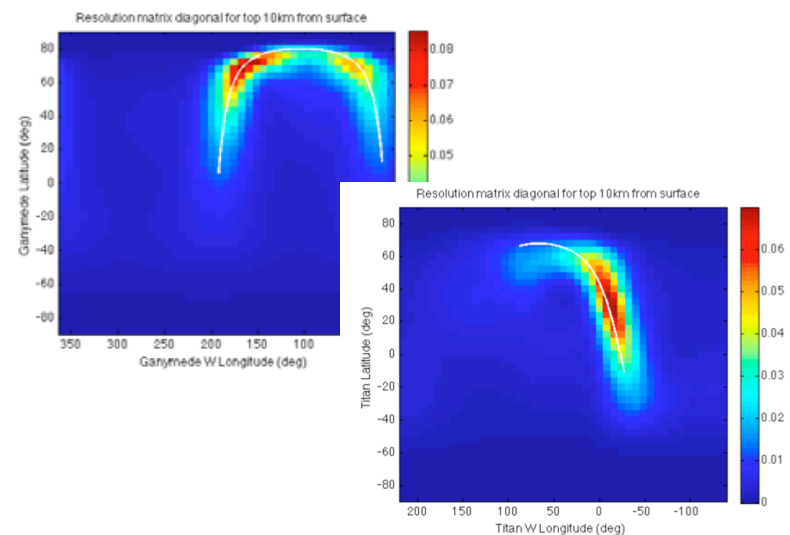


# Intro via examples

*Ocean bottom (“geoacoustic”) inversion: estimate seafloor properties from sonar in water*



*Radio doppler gravimetry of planetary bodies: estimate density of icy moon interiors*



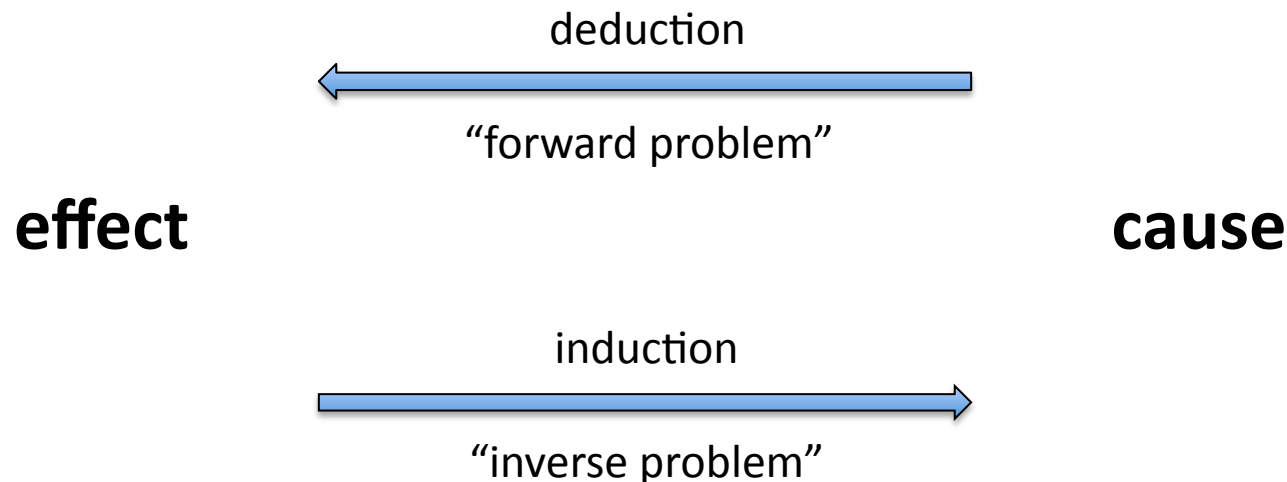
# Deduction vs. Induction

*Note the common theme in those examples (but there are others):  
inferring properties of interior from measurements on an exterior.*

**predicted\_data = somefunction( model\_of\_interest )**

gravity( $x_i$ )  
traveltime(depth <sub>$i$</sub> )  
waveintensity( $x_i, t_j$ )  
dopplerfreq( $t_j$ )  
chemconcentration( $x_i, t_j$ )

density( $x, z$ )  
wavespeed( $z$ )  
temperature( $z, t$ )  
chemsrcleakage( $t$ )  
etc...



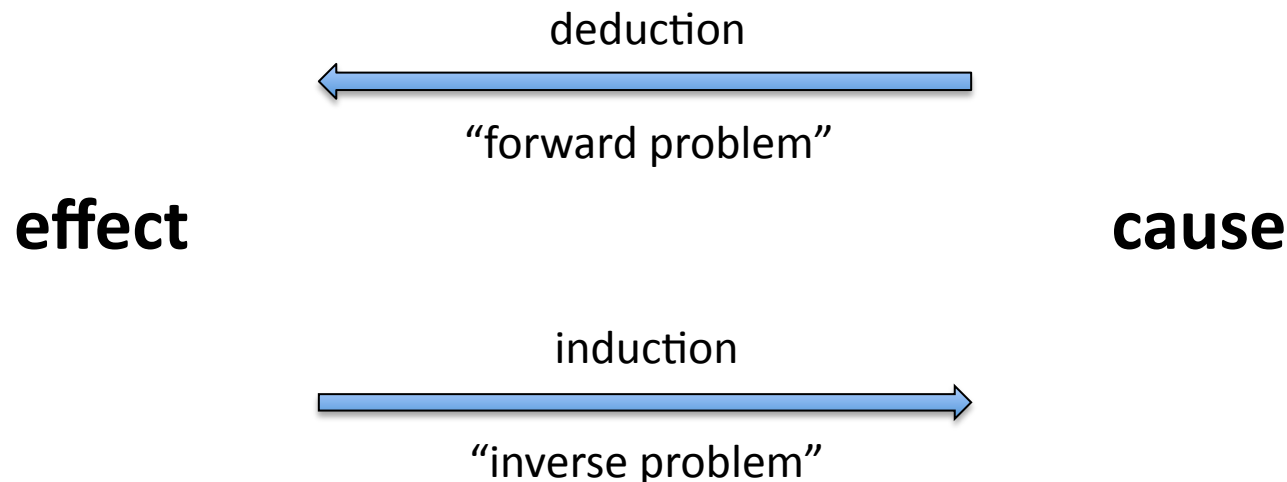
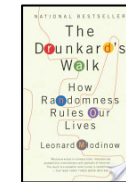
# Probability vs. Statistics

*Stemming straight from the difference between deduction and induction*

In these cases it is the latter scenario that is more often useful in life: outside situations involving gambling, we are not normally provided with theoretical knowledge of the odds but rather must estimate them after making a series of observations. Scientists, too, find themselves in this position: they do not generally seek to know, given the value of a physical quantity, the probability that a measurement will come out one way or another but instead seek to discern the true value of a physical quantity, given a set of measurements.

I have stressed this distinction because it is an important one. It defines the fundamental difference between probability and statistics: the former concerns predictions based on fixed probabilities; the latter concerns the inference of those probabilities based on observed data.

- Leonard Mlodinow  
*The Drunkard's Walk*  
(highly recommended!)

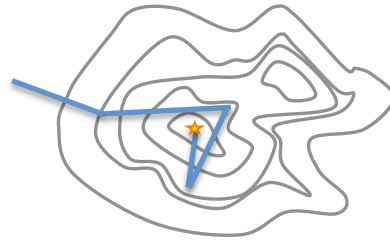




# Frequentist vs. Bayesian

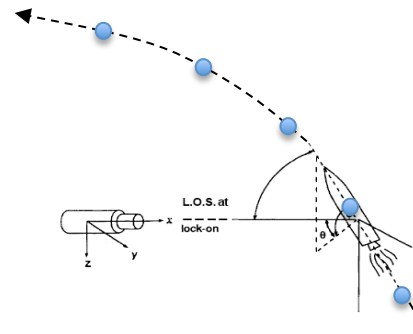
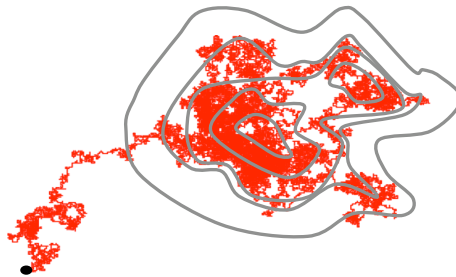
*A debate raging for 200+ years in the statistics community*

- **Frequentists** define probability in terms of frequency of repeatable events.  
So one can't know anything about model before the event/experiment.  
*Most common tool* – linear (or iteratively linear) approach to problem.



The ESS class concentrates on frequentist tools with iteratively linear solution techniques – you can only fit so much into one quarter...

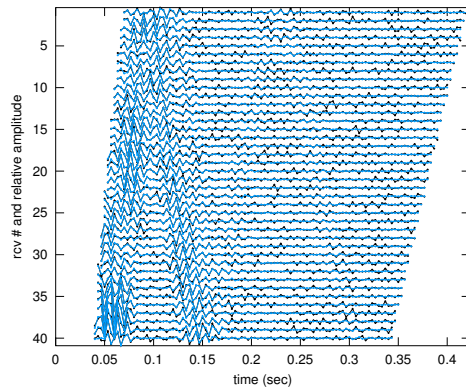
- **Bayesians** define probability in terms of degree of belief.  
So one *can* know about the model before the event/experiment.  
*Common tools* – fancy, computationally-heavy MCMC inversion, but can do linear/iteratively-linear problems too, and also can do filters (eg Kalman).



# Data vs. Model

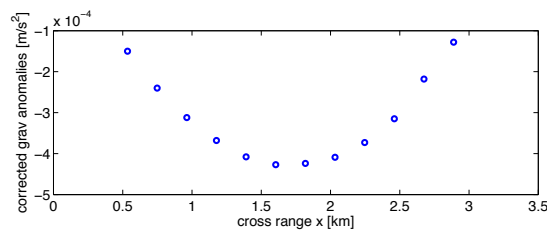
*Note the different sets of X & Y axes in the two spaces.*

**predicted\_data = somefunction( model\_of\_interest )**

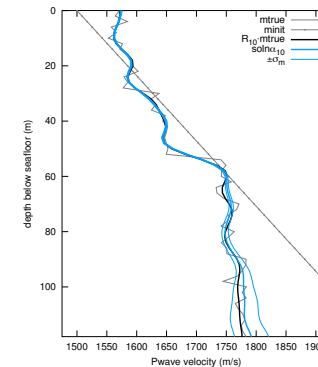


**data space**

(measurements & predictions of them)



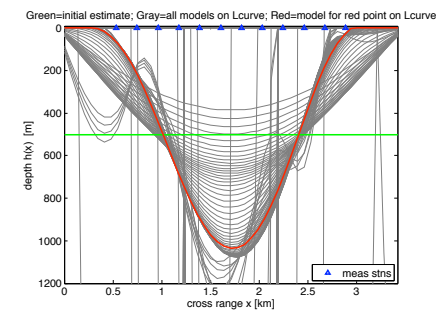
*Seafloor  
acoustic  
example*



**model space**

(what we really want to know)

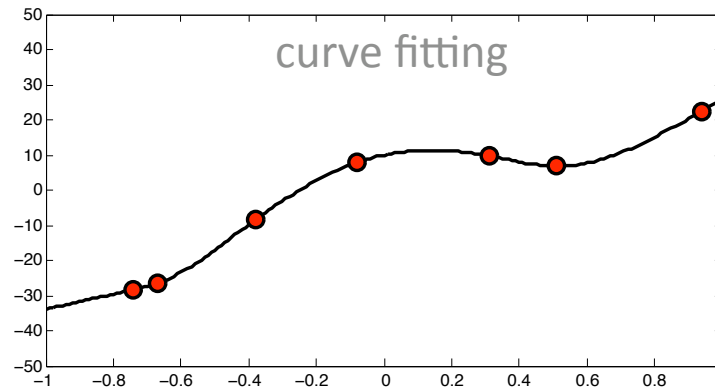
*Glacier  
gravimetry  
example*



# Data vs. Model

*Same set of X & Y axes in the two spaces in this special case.*

```
predicted_data = somefunction( model_of_interest )
```



**data space**

(measurements & predictions of them)

**model space**

(what we really want to know)

These two spaces are the **same** in the special case of **curve fitting**.  
But that's only a special case.

# Math vs. Earth Science

*Why not just @\$% flip it around mathematically and call it done?!*

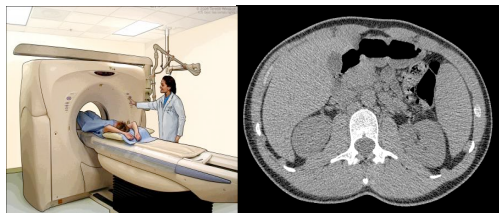
$$\begin{array}{lcl} \text{predicted\_data} & = & \text{somefunction}(\text{model\_of\_interest}) \\ d(s) & & m(x) \end{array}$$

$$\begin{array}{lcl} \text{model\_of\_interest} & = & \text{somefunction}^{-1}(\text{measured\_data}) \\ m(x) & & d(s) \end{array}$$

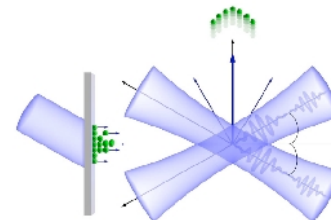
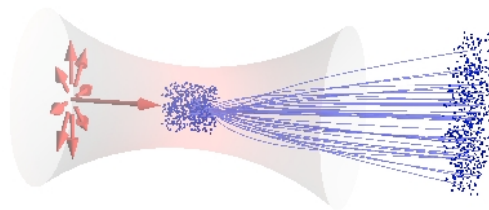
$$d(s_i)$$

$$d(s_i) + \text{noise}$$

Sometimes you can, e.g. :



CT scans (Radon transform)

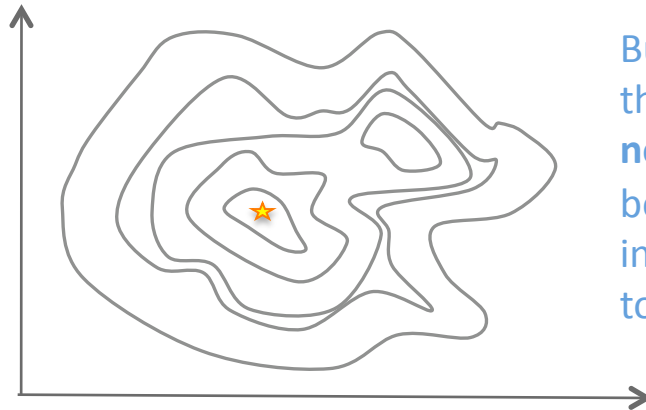


nuclear scattering experiments (inverse scattering theory)

# Math vs. Earth Science

*Why not just @\$% flip it around mathematically and call it done?!*

`predicted_data = somefunction( model_of_interest )`



But in many Earth science problems, the **geometric coverage** is lousy and the **noise** is great enough that  $\text{somefunction}^{-1}()$  becomes hopelessly unstable. We must use instead approaches related to optimization to do the inversion.

**Central issues for inverse problem solutions:**

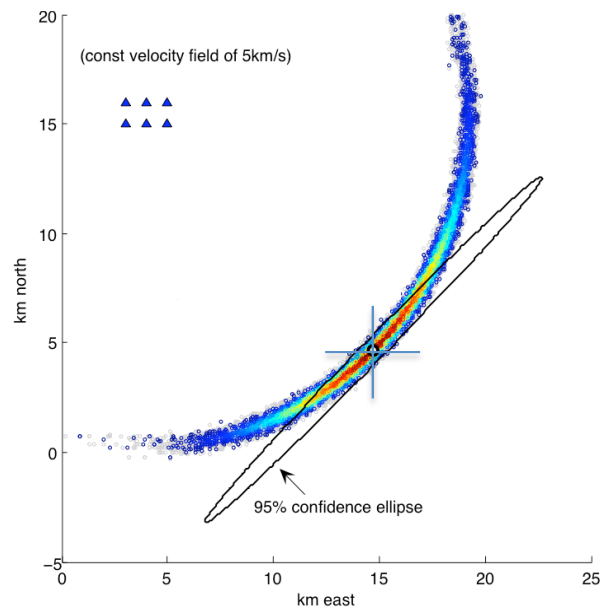
- **existence**
- **uniqueness**
- **stability**
- **uncertainty**



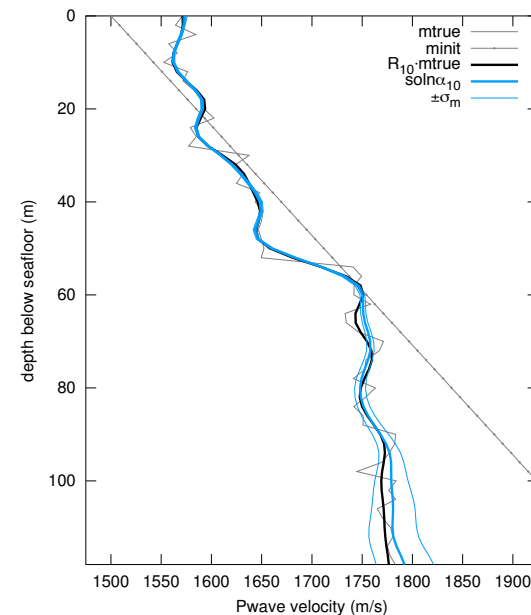
# Parameter estimation vs. inversion

**parameter estimation:** solve for a handful of discrete values

**inversion:** solve for a continuous function (be it 1D, 2D, etc.) – much more involved (although it *uses* parameter estimation)



param est: find src x,y



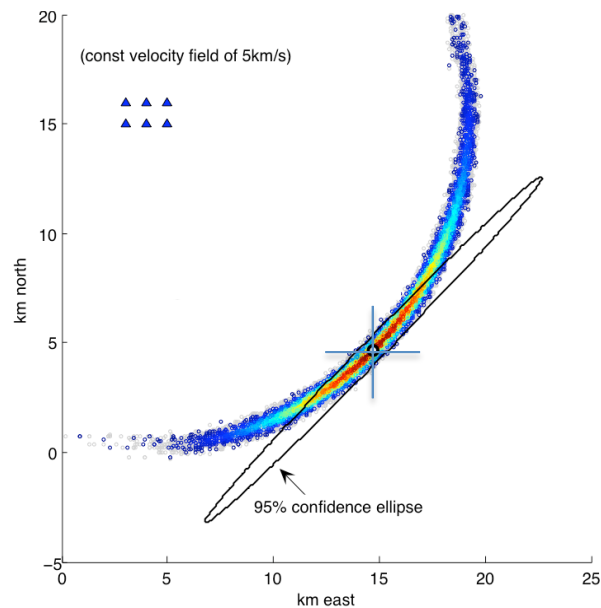
inv: find vel(z)

but actually...

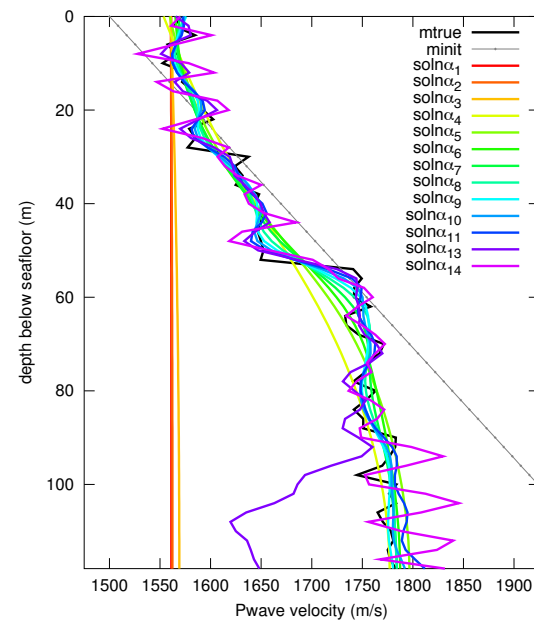
# Parameter estimation vs. inversion

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param est: find src x,y

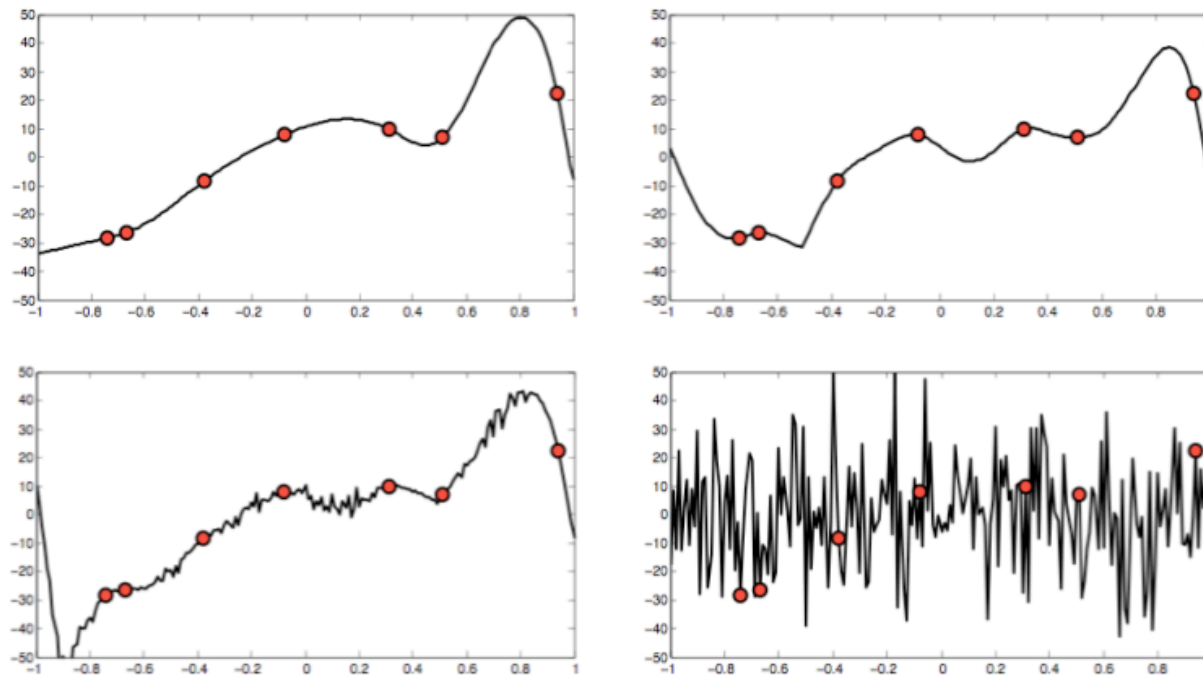


inv: uh-oh, **many** curves  
produce predictions that  
fit the data

# Parameter estimation vs. inversion

**parameter estimation:** solve for a handful of discrete values

**inversion:** solve for a continuous function (be it 1D, 2D, etc.) – much more involved (although it *uses* parameter estimation)



An intuitive example via curve-fitting problem:

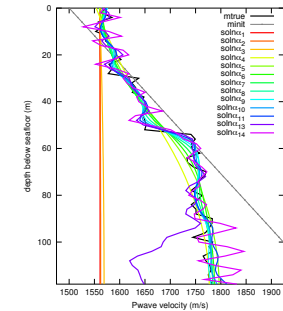
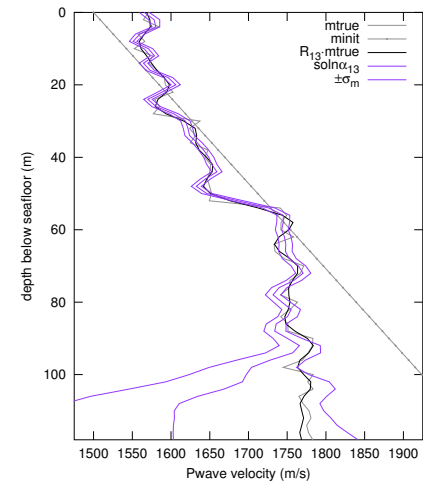
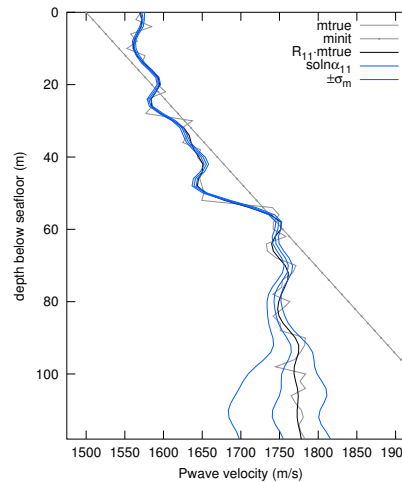
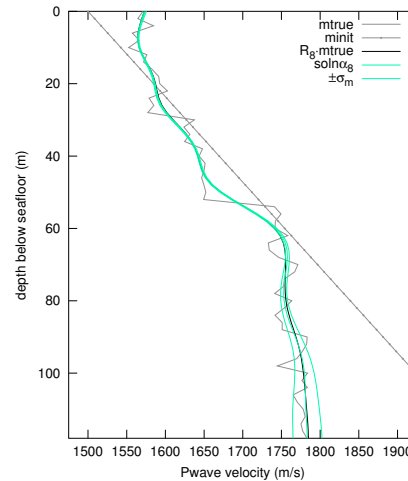
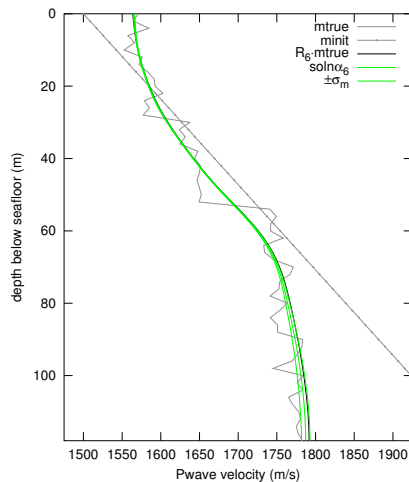
inv: uh-oh, **many** curves  
produce predictions that  
fit the data

# Uncertainty vs. Resolution

*X-axis “smearing”*

*Y-axis “smearing”*

Four of those many curves now shown separately, with their uncertainties included around them.



*(Uncertainties are so narrow in top halves only because this was from a highly idealized synthetic problem.)*

**Choose what you want:**

**Higher-res solutions have larger uncertainties, lower-res solutions have smaller uncertainties.**

# Linear vs. Nonlinear

`predicted_data` = `somefunction( model_of_interest )`  
contained in the vector  $\mathbf{d}$  parameterized by the vector  $\mathbf{m}$

**Linear problems:** scalability and superposition;  
Gaussians map to Gaussians;  
Computes fast – jump to solution in one step;

$$\mathbf{d} = \mathbf{F} \mathbf{m}$$

**Nonlinear problems:** more general (and more common!);  
Uniqueness, stability, uncertainty take **MUCH** more effort & interpretation;  
Slower – use sequence of linear subproblems, or use many MC samples.

$$\mathbf{d} = \mathbf{f} ( \mathbf{m} )$$



# The Class: ESS 523

- **Overall:** learn how to do linear problems, then set up your nonlinear problem as a sequence of linear ones.
- Will extensively use **Matlab** or Octave (free/awesome GNU clone of Matlab)
- **Recommended Prerequisite background:**
  - Basic probability & statistics concepts -
    - e.g. mean, std dev, variance, covariance, correlation
  - Linear algebra -
    - e.g. matrix/vector arithmetic, transpose, inverse, null space, rank, condition number, eigenvalues/vectors, under/over-determined probs
  - Fourier transforms (time/space  $\leftrightarrow$  frequency)
  - Some idea of connection between the class and your research
- No tests, but **weekly labs and a class project** based on your research

# Shameless plug

<http://staff.washington.edu/aganse>

(also linked via ESS and APL directory pages)

## Andy Ganse's Geophysical Inverse Theory Resources Page

Andy Ganse, Applied Physics Laboratory, University of Washington, Seattle

Home / Inverse Theory Resources /

- Home
- C.V.
- Current Research & Pubs
- Inverse Theory Resources**
- 2004 Summer School
- Side Interests
- My Bookshelf
- Downloads
- Goofy Stuff

**Some handy quick links:**

[UW \(Seattle\) Math Dept Inverse Problems seminars](#)  
(you know how those pure mathematicians are; be sure to keep them honest by occasionally bringing up questions about noise and stability!)

[Inverse Problems journal](#)

[Google Scholar \(academic paper\)](#)

**A growing list of recommended textbooks and helpful papers, Q&A list, related web links, and lecture notes, all on aspects of geophysical inverse theory.**

- **Recommended reading**
  - **Textbooks:**  
(Note also my "favorite textbooks" list on my [Books/Reading List](#) webpage, which includes the below books on inverse theory along with others on different topics in geophysics and math.)
    - [Parameter Estimation and Inverse Problems](#), by Richard Aster, Brian Borchers, Clifford Thurber.  
Note also the [homepage](#) for this book which includes errata.  
For beginners to inversion, this book is strongly recommended above the others; there are plenty very useful books on the topic, but this one really gets you up to speed in the subject fast with great hands-on Matlab examples. Then, after you're more familiar with the material, go back and reread the book again - there are tons of handy comparisons between methods with references to deeper treatment of the individual methods elsewhere.  
limitations.
    - [Inverse Problem Theory](#)  
Very well written book with useful comparisons between copy of this book on his y can afford it.)
    - [Rank-Deficient and Discrete](#)  
Hansen.  
Very well written book co (but not all) can be found free unlike this book!
    - [Geophysical Inverse The](#)  
A classic text that is very concepts, but injects with Gram matrix / represent parameters as there are data points, and requires numerical integration for many real-world problems.

- Recommended textbooks
- Recommended journal papers
- Links to software and other web resources
- Lecture notes and labs from the inverse theory class I TA'd.

# Another shameless plug

<http://staff.washington.edu/aganse>


(also linked via ESS and APL directory pages)

## Nonlinear Filtering Examples from Gelb

Andy Ganse, Applied Physics Laboratory, University of Washington, Seattle

Home / Current Research & Pubs /

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- C.V.
- Current Research & Pubs
- Publications & Abstracts
- Filter examples
- Inverse Theory Resources
- Side Interests
- My Bookshelf
- Downloads
- Goofy Stuff

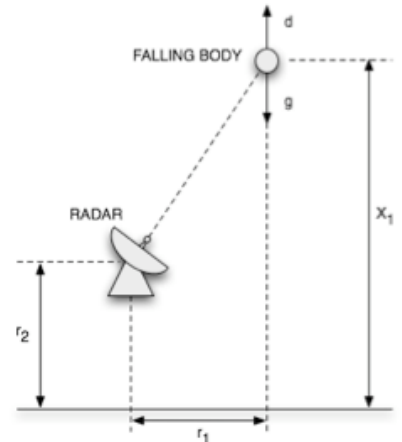


### A Matlab script to recompute the nonlinear tracking filter examples 6.1-3 in Gelb

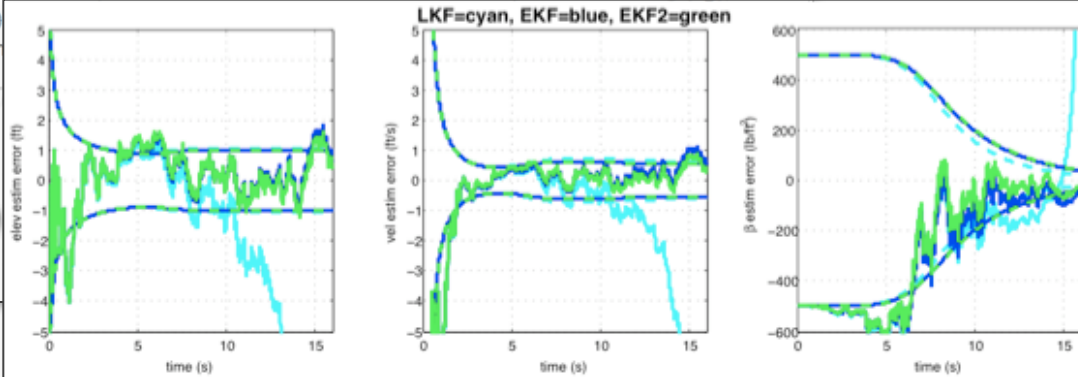
My inverse theory research relies on concepts from recursive filters, so I had to take some time speed on those. A classic textbook for this is *Applied Optimal Estimation*, edited by Gelb (1974) of that book are two simple radar tracking examples (6.1-2 and 6.1-3) which demonstrate several filters. I've programmed up those examples into a Matlab script called [gravdragdemo.m](http://staff.washington.edu/aganse/gravdragdemo.m) and added filters to compare and contrast them in both linear and nonlinear cases.

These examples use radar ranging to estimate the elevation, downward velocity, and drag coefficient of a falling body as functions of time. These three values are collected into a  $3 \times 1$  vector called the state  $\underline{x}$  again a function of time. The two examples are related: example 6.1-3 has a 2D arrangement of nonlinear measurements with respect to  $\underline{x}$ . Example 6.1-2 is a special case of 6.1-3 in which the measurements are collapsed to 1D by letting  $r_1$  and  $r_2$  shrink to zero, causing the measurement relation to become linear with respect to  $\underline{x}$ . The dynamics of both examples in the book are nonlinear because they include air drag ( $x_3$ ), which depends on velocity ( $x_2$ ) (see example 6.1-2 for the linear case) and example 6.1-3 (nonlinear measurements case) and example 6.1-2 (linear measurements case).

#### Example 6.1-3: nonlinear measurements, nonlinear dynamics



Redrawn from figure 6.1-5, *Applied Optimal Filter*, ed. Gelb, The Analytic Sciences Corporation, 1994.



Example 6.1-3: nonlinear measurements, nonlinear dynamics

FALLING BODY

elev estim error (ft)

time (s)

vel estim error (ft/s)

time (s)

$\beta$  estim error (lb/ft<sup>2</sup>)

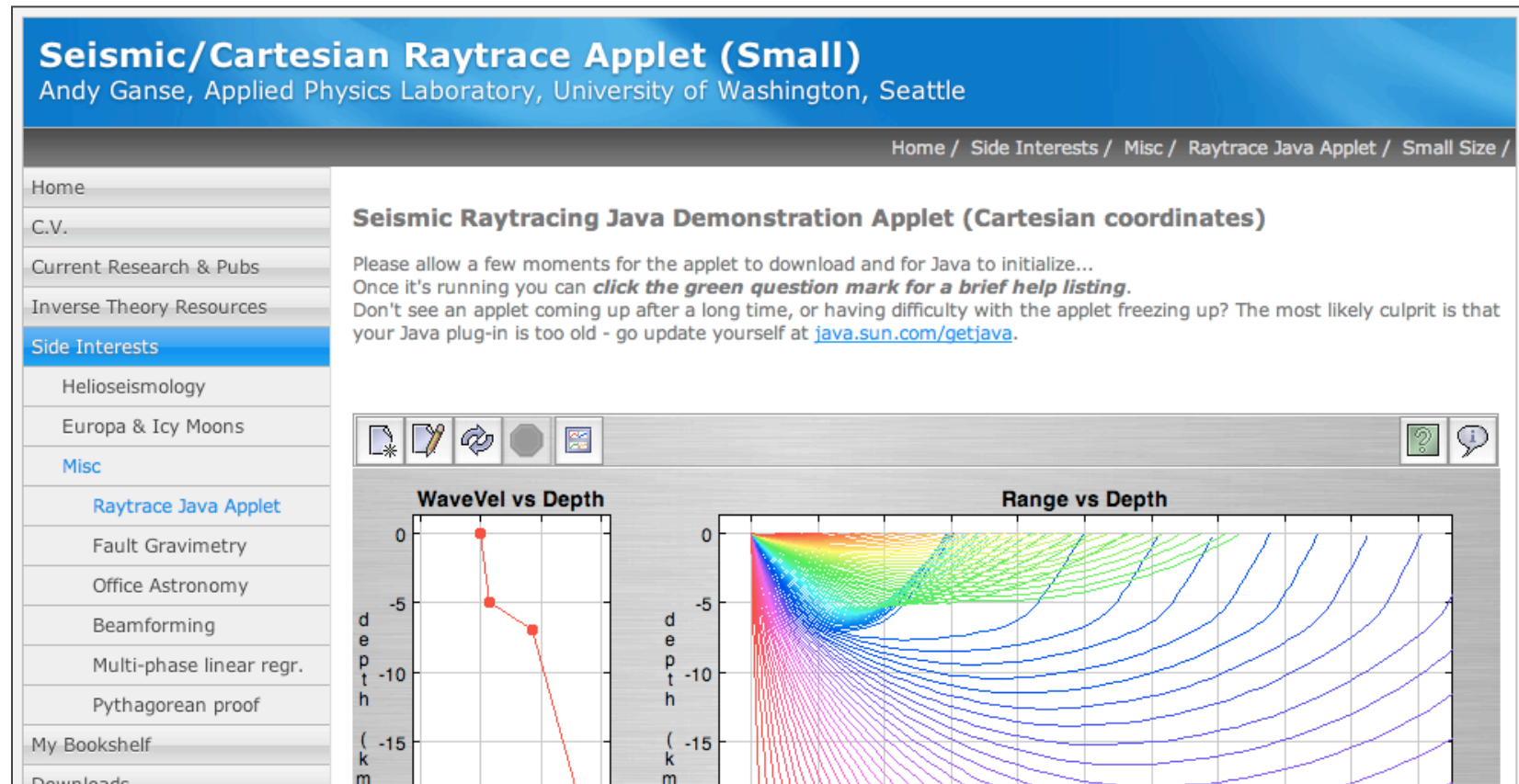
time (s)

LKF=cyan, EKF=blue, EKF2=green

# Fortunately, not too many shameless plugs...

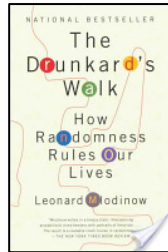
<http://staff.washington.edu/aganse>

(also linked via ESS and APL directory pages)

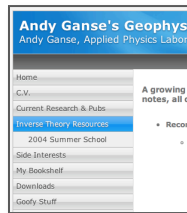


- Enter wave velocity profiles and watch the rays go!
- Spherical geometry one available too...

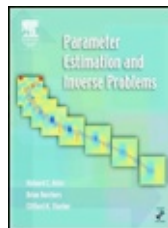
# Recommended reading



Really fantastic popular book re probability and statistics:  
**The Drunkard's Walk,**  
by Leonard Mlodinow



**My website** (of course!) – pages on inverse theory resources, linear and nonlinear filter tutorial, ray-tracing, and much more.  
<http://staff.washington.edu/aganse>



The best frequentist inverse theory textbook:  
**Parameter Estimation and Inverse Theory,**  
by Aster, Borchers, Thurber



The best Bayesian inverse theory textbook:  
**Inverse Problem Theory and Model Parameter Estimation,**  
by Albert Tarantola (available free online!)