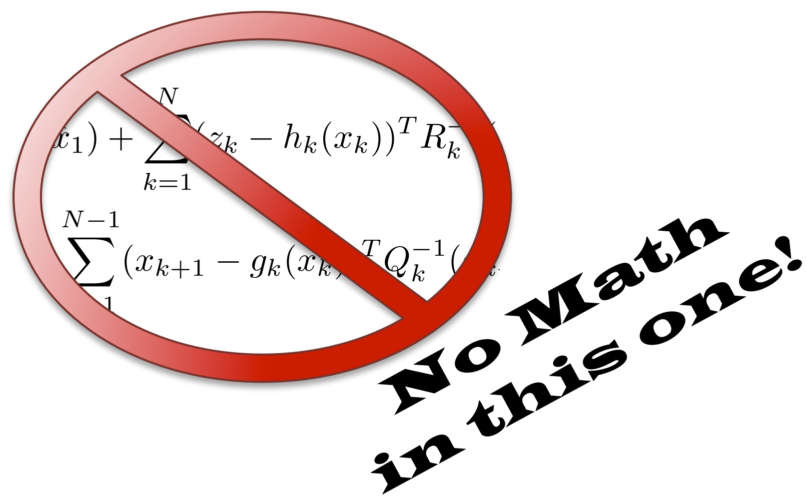
A Conceptual Introduction to Geophysical Inversion

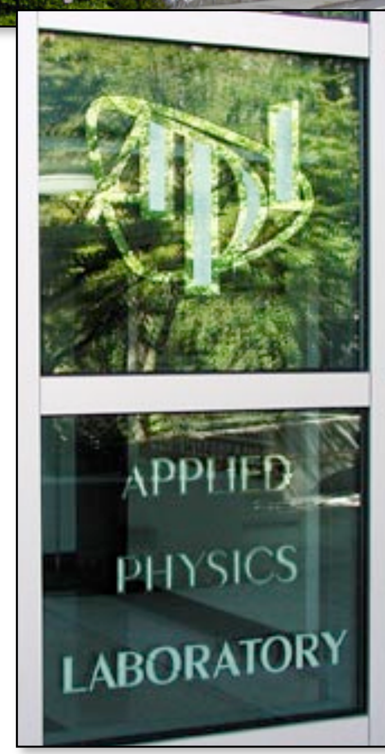
Andy Ganse
ESS & Applied Physics Lab
University of Washington

12 Mar 2012



A little about APL

(my stomping ground…)



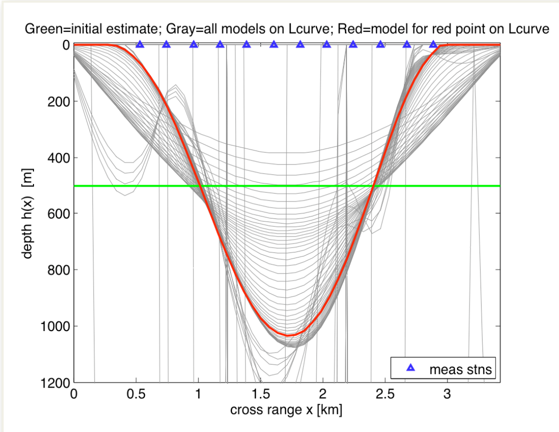
A conceptual / non-technical talk

A terminology extravaganza in a sequence of dichotomies…

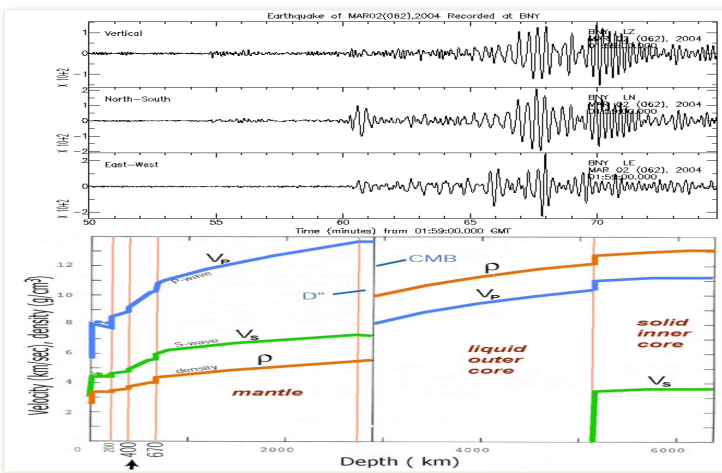
- Intro via examples
- Data vs. Model
- Deduction vs. Induction
- Probability vs. statistics
- Frequentist vs. Bayesian
- Math vs. Earth Science
- Parameter estimation vs. inversion
- Uncertainty vs. Resolution
- Linear vs. Nonlinear
- Recommended reading

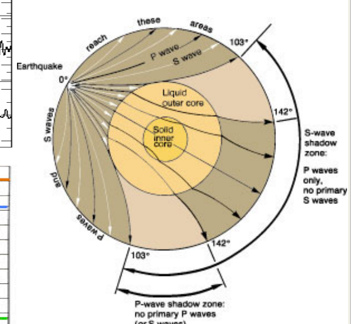
Intro via examples

Glacier gravimetry: estimate glacier cross-section from gravity measurements



Global seismic inversion: estimate Earth's interior wavespeeds & densities from EQ seismograms

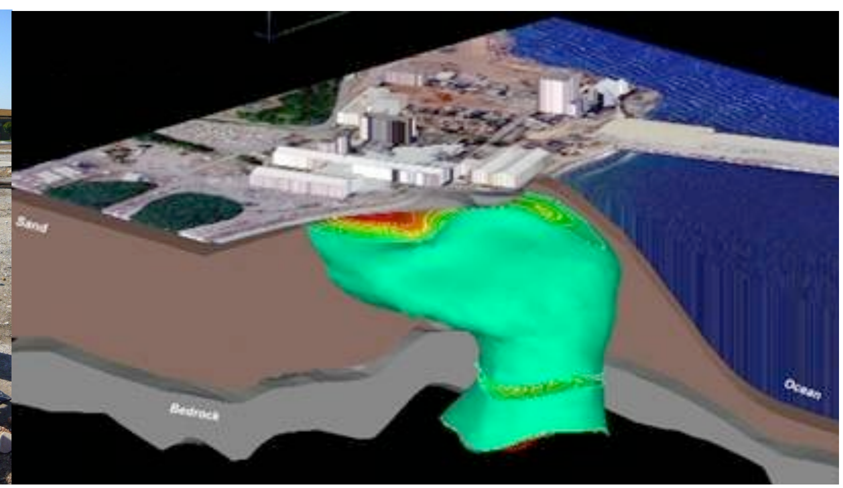




Intro via examples

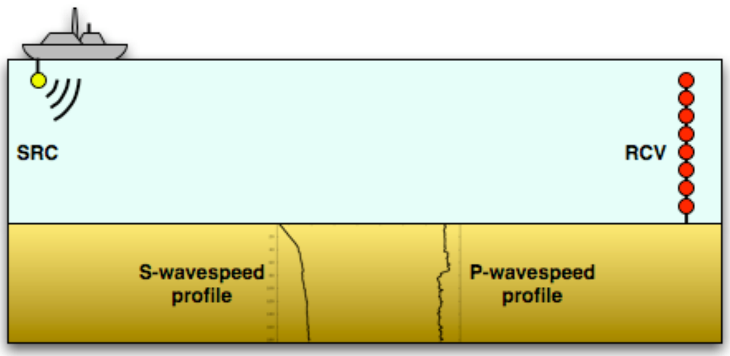
Computerized Tomography (CT) scans: estimate 3D body interior densities from Xray atten

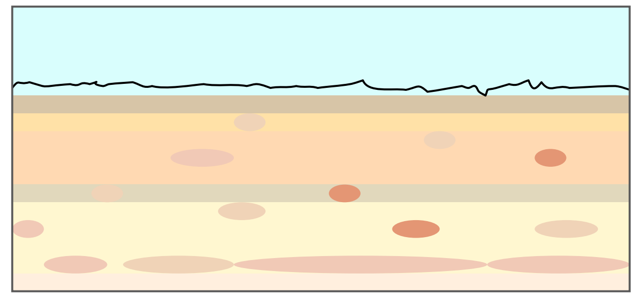
Groundwater contamination: estimate source leakage function from groundwater samplings



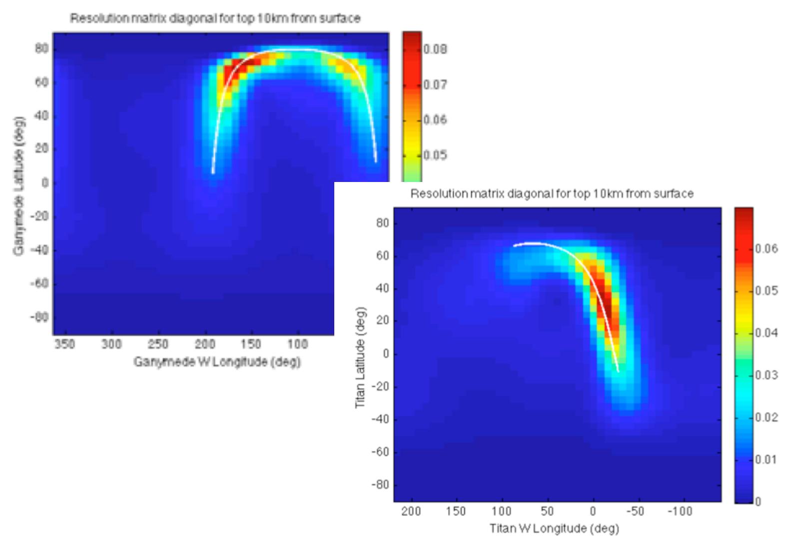
Intro via examples

Ocean bottom ("geoacoustic") inversion: estimate seafloor properties from sonar in water





Radio doppler gravimetry of planetary bodies: estimate density of icy moon interiors



Deduction vs. Induction

Note the common theme in those examples (but there are others):
inferring properties of interior from measurements on an exterior.

```
predicted_data  =  somefunction(  model_of_interest  )
```

gravity(x_i)
traveltime($depth_i$)
waveintensity(x_i,t_j)
dopplerfreq(t_j)
chemconcentration($x_{i,}t_j$)

density(x,z)
wavespeed(z)
temperature(z,t)
chemsrcleakage(t)
etc…

deduction

← “forward problem”

effect **cause**

induction

→ “inverse problem”

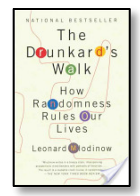
Probability vs. Statistics

Stemming straight from the difference between deduction and induction

In these cases it is the latter scenario that is more often useful in life: outside situations involving gambling, we are not normally provided with theoretical knowledge of the odds but rather must estimate them after making a series of observations. Scientists, too, find themselves in this position: they do not generally seek to know, given the value of a physical quantity, the probability that a measurement will come out one way or another but instead seek to discern the true value of a physical quantity, given a set of measurements.

I have stressed this distinction because it is an important one. It defines the fundamental difference between probability and statistics: the former concerns predictions based on fixed probabilities; the latter concerns the inference of those probabilities based on observed data.

- Leonard Mlodinow
The Drunkard's Walk
(highly recommended!)



deduction

← "forward problem"

effect **cause**

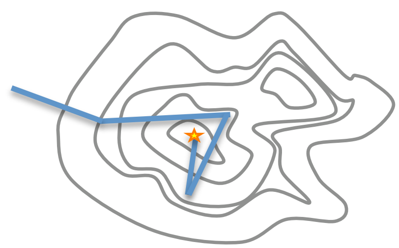
induction

→ "inverse problem"

Frequentist vs. Bayesian

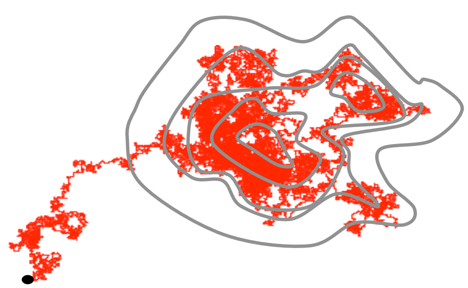
A debate raging for 200+ years in the statistics community

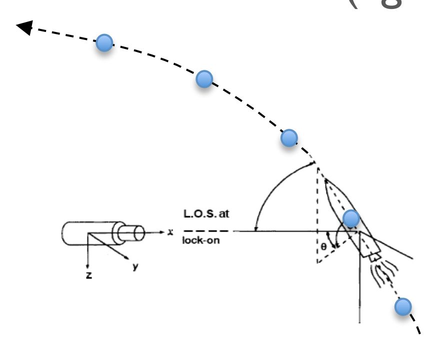
- **Frequentists** define probability in terms of <u>frequency of repeatable events</u>.
  So one can't know anything about model before the event/experiment.
  Most common tool – linear (or iteratively linear) approach to problem.



The ESS class concentrates on frequentist tools with iteratively linear solution techniques – you can only fit so much into one quarter…

- **Bayesians** define probability in terms of <u>degree of belief</u>.
  So one *can* know about the model before the event/experiement.
  Common tools – fancy, computationally-heavy MCMC inversion, but can do linear/iteratively-linear problems too, and also can do filters (eg Kalman).

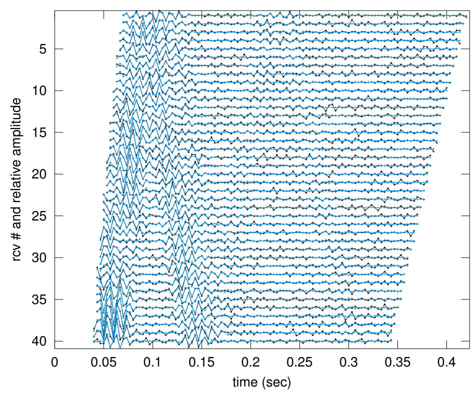




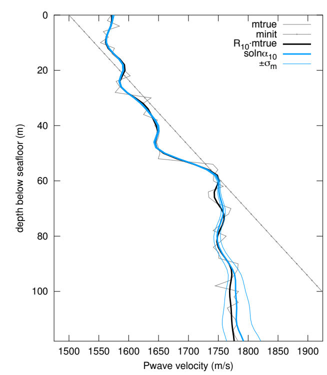
Data vs. Model

Note the different sets of X & Y axes in the two spaces.

```
predicted_data  =  somefunction(  model_of_interest  )
```



Seafloor acoustic example

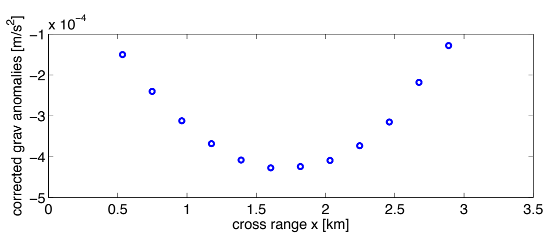


data space

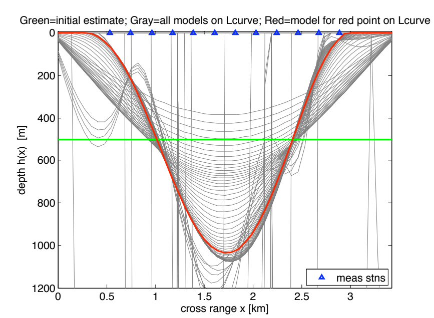
(measurements & predictions of them)

model space

(what we really want to know)



Glacier gravimetry example

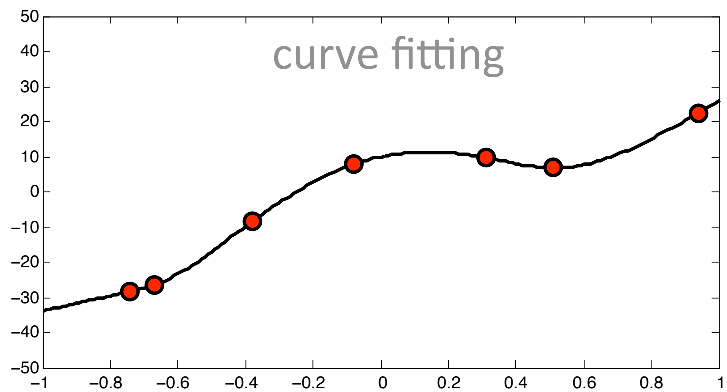


Ganse

Data vs. Model

Same set of X & Y axes in the two spaces in this special case.

```
predicted_data  =  somefunction(  model_of_interest  )
```



data space
(measurements & predictions of them)

model space
(what we really want to know)

These two spaces are the **same** in the special case of **curve fitting**.
But that's only a special case.

Math vs. Earth Science

Why not just @#$% flip it around mathematically and call it done?!

predicted_data = somefunction(model_of_interest)

$d(s)$ $m(x)$

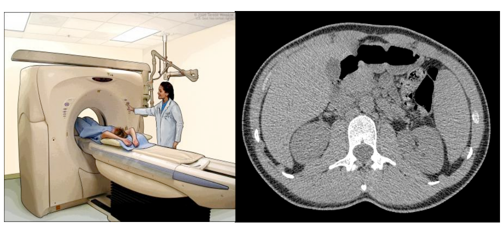
model_of_interest = somefunction⁻¹(measured_data)

$m(x)$ $d(s)$

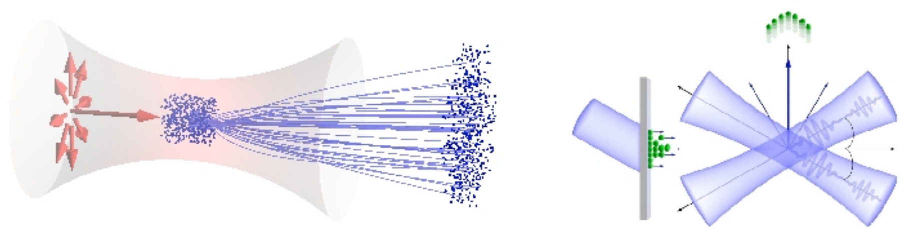
$d(s_i)$

$d(s_i)$ + noise

Sometimes you can, e.g. :



CT scans (Radon transform)

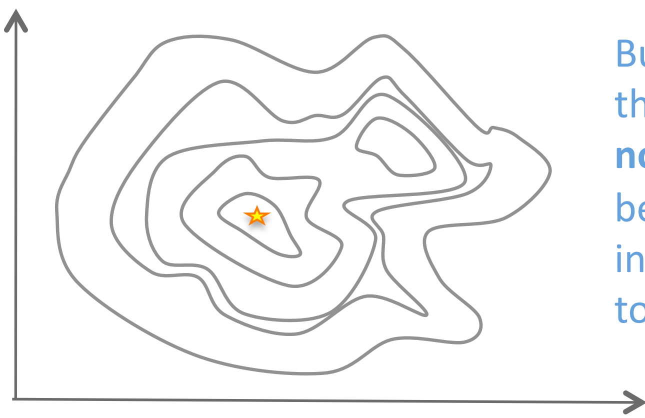


nuclear scattering experiments (inverse scattering theory)

Math vs. Earth Science

Why not just @#$% flip it around mathematically and call it done?!

```
predicted_data  =  somefunction(  model_of_interest  )
```



But in many Earth science problems, the **geometric coverage** is lousy and the **noise** is great enough that somefunction^{-1}() becomes hopelessly unstable. We must use instead approaches related to optimization to do the inversion.

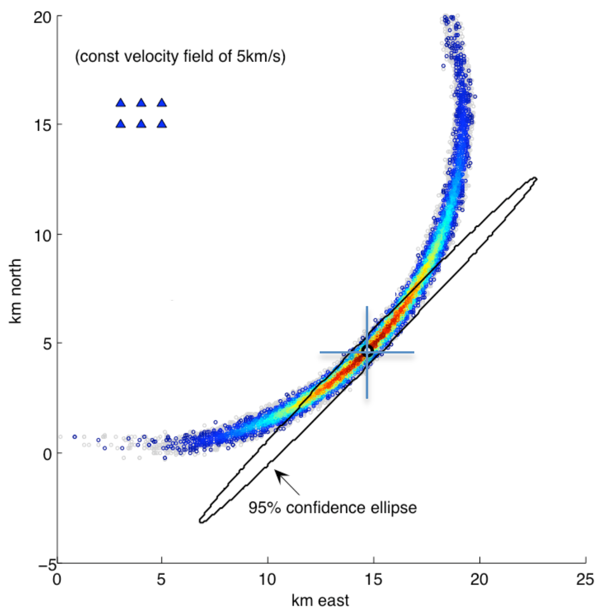
Central issues for inverse problem solutions:

- existence
- uniqueness
- stability
- uncertainty

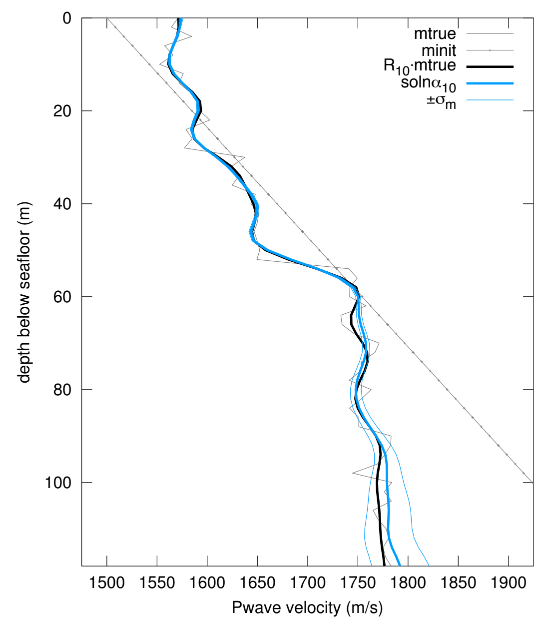
Parameter estimation vs. inversion

parameter estimation: solve for a handful of discrete values

inversion: solve for a continuous function (be it 1D, 2D, etc.) – much more involved (although it *uses* parameter estimation)



param est: find src x,y



inv: find vel(z)

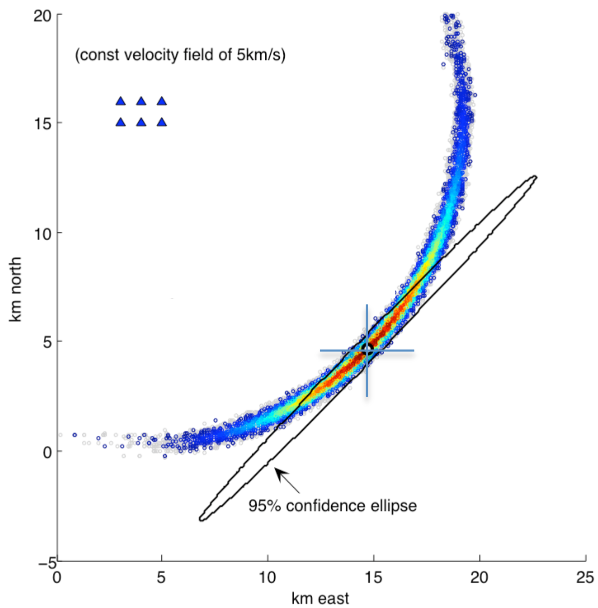
Ganse

but actually…

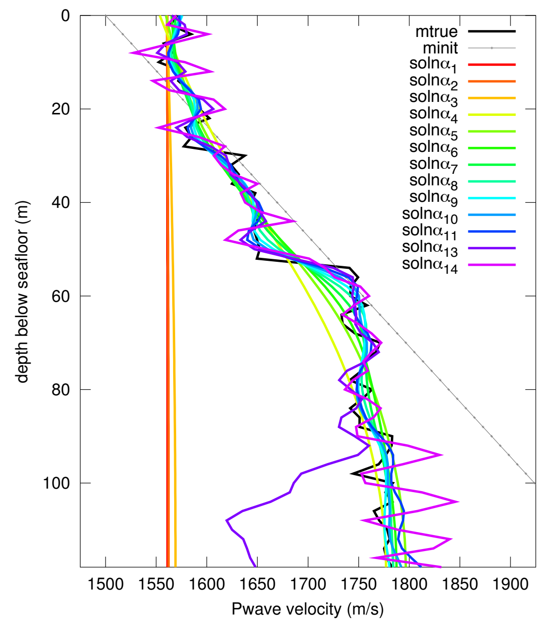
Parameter estimation vs. inversion

parameter estimation: solve for a handful of discrete values

inversion: solve for a continuous function (be it 1D, 2D, etc.) – much more involved (although it *uses* parameter estimation)



param est: find src x,y

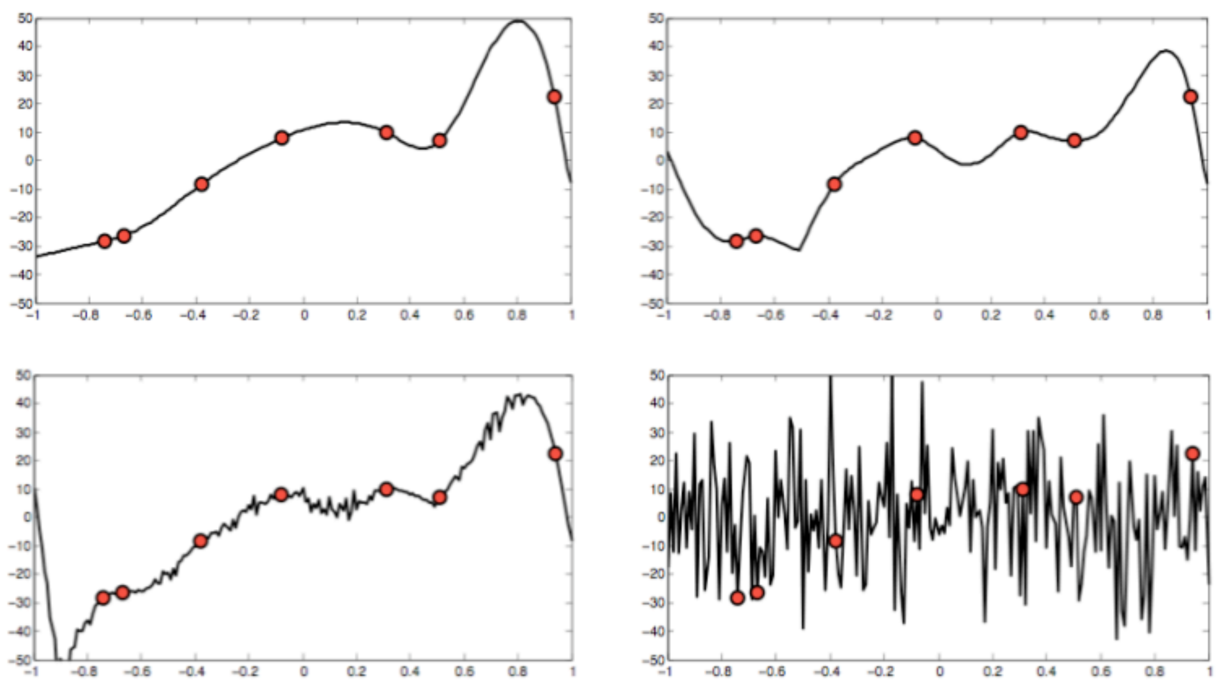


inv: uh-oh, **many** curves produce predictions that fit the data

Parameter estimation vs. inversion

parameter estimation: solve for a handful of discrete values

inversion: solve for a continuous function (be it 1D, 2D, etc.) – much more involved (although it *uses* parameter estimation)

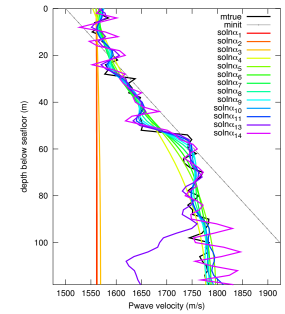


An intuitive example via curve-fitting problem:

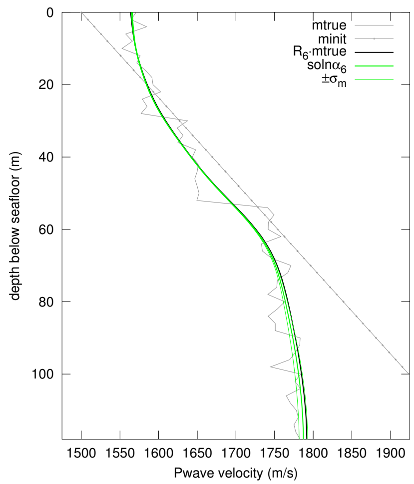
inv: uh-oh, **many** curves produce predictions that fit the data

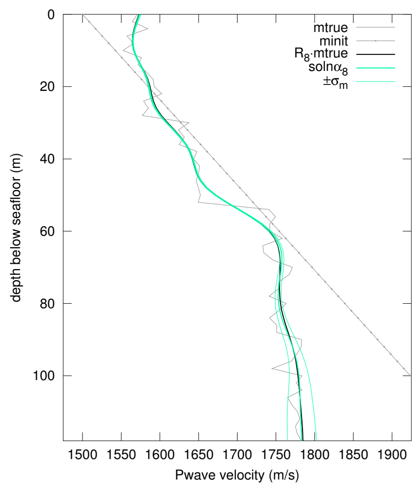
Uncertainty vs. Resolution

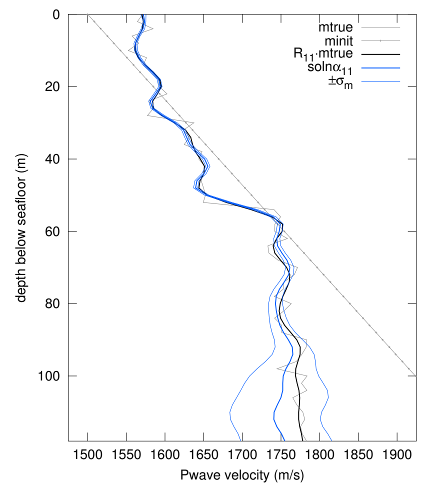
X-axis "smearing" *Y-axis "smearing"*

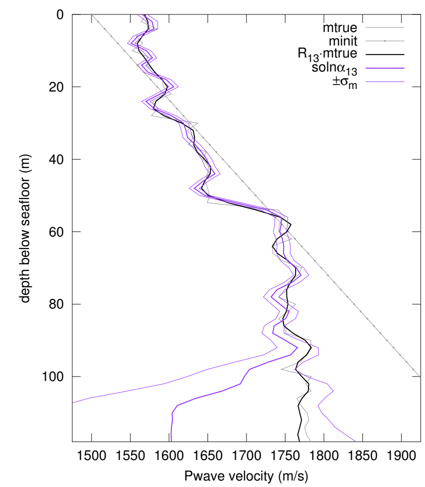


Four of those many curves now shown separately, with their uncertainties included around them.









(Uncertainties are so narrow in top halves only because this was from a highly idealized synthetic problem.)

Choose what you want:

Higher-res solutions have larger uncertainties, lower-res solutions have smaller uncertainties.

Linear vs. Nonlinear

```
predicted_data  =  somefunction(  model_of_interest  )
```

contained in the vector **d**

parameterized by the vector **m**

Linear problems: scalability and superposition;
Gaussians map to Gaussians;
Computes fast – jump to solution in one step;

$$\mathbf{d} = \mathbf{F}\,\mathbf{m}$$

Nonlinear problems: more general (and more common!);
Uniqueness, stability, uncertainty take **MUCH** more effort & interpretation;
Slower – use sequence of linear subproblems, or use many MC samples.

$$\mathbf{d} = \mathbf{f}(\mathbf{m})$$

The Class: ESS 523

- **Overall**: learn how to do linear problems, then set up your nonlinear problem as a sequence of linear ones.

- Will extensively use **Matlab** or Octave (free/awesome GNU clone of Matlab)

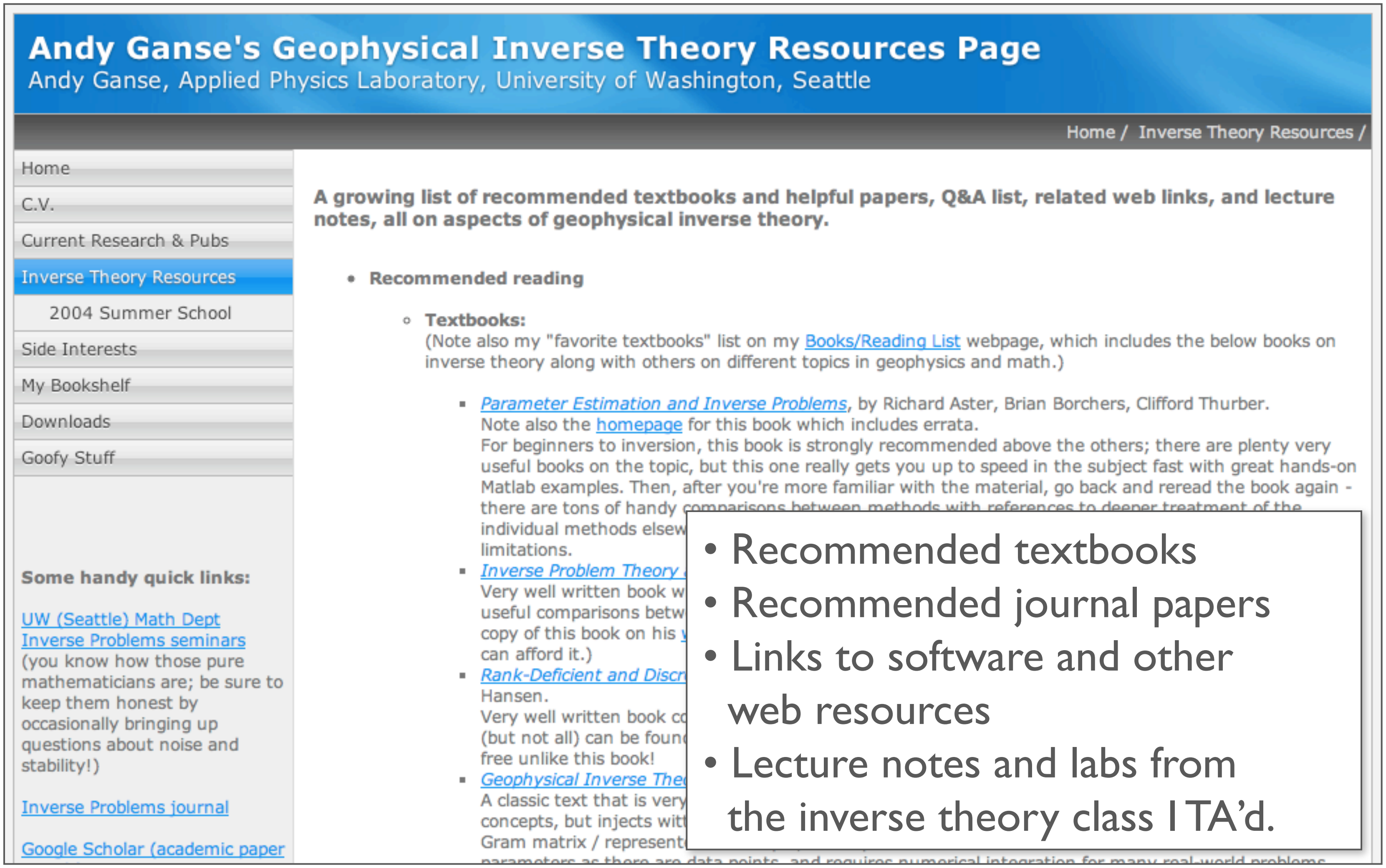
- **Recommended Prerequisite background**:
    - Basic probability & statistics concepts -
        - e.g. mean, std dev, variance, covariance, correlation
    - Linear algebra -
        - e.g. matrix/vector arithmetic, transpose, inverse, null space, rank, condition number, eigenvalues/vectors, under/over-determined probs
    - Fourier transforms (time/space ←→ frequency)
    - Some idea of connection between the class and your research

- No tests, but **weekly labs and a class project** based on your research

Shameless plug

http://staff.washington.edu/aganse

(also linked via ESS and APL directory pages)

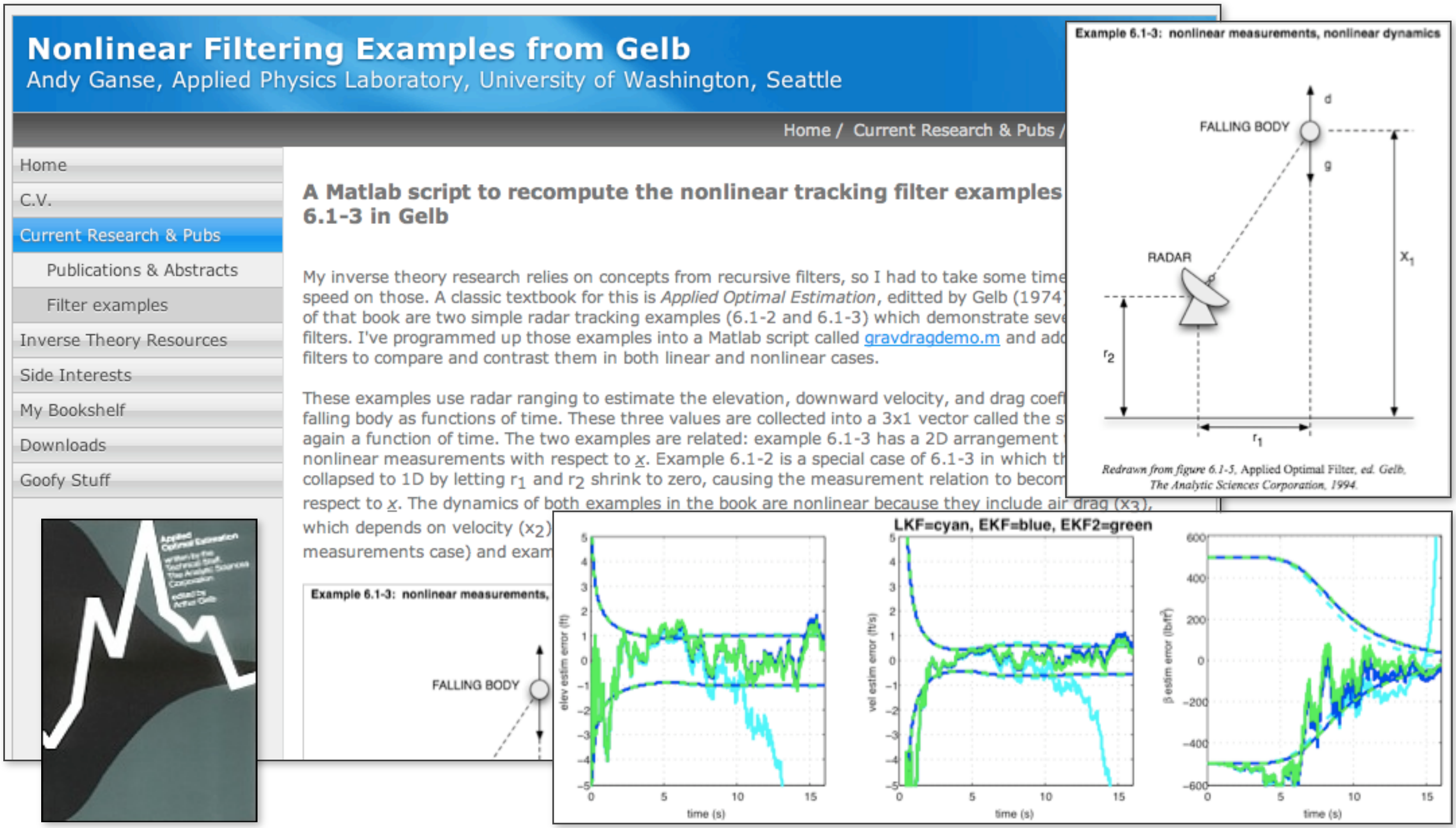


- Recommended textbooks
- Recommended journal papers
- Links to software and other web resources
- Lecture notes and labs from the inverse theory class I TA'd.

Another shameless plug

http://staff.washington.edu/aganse

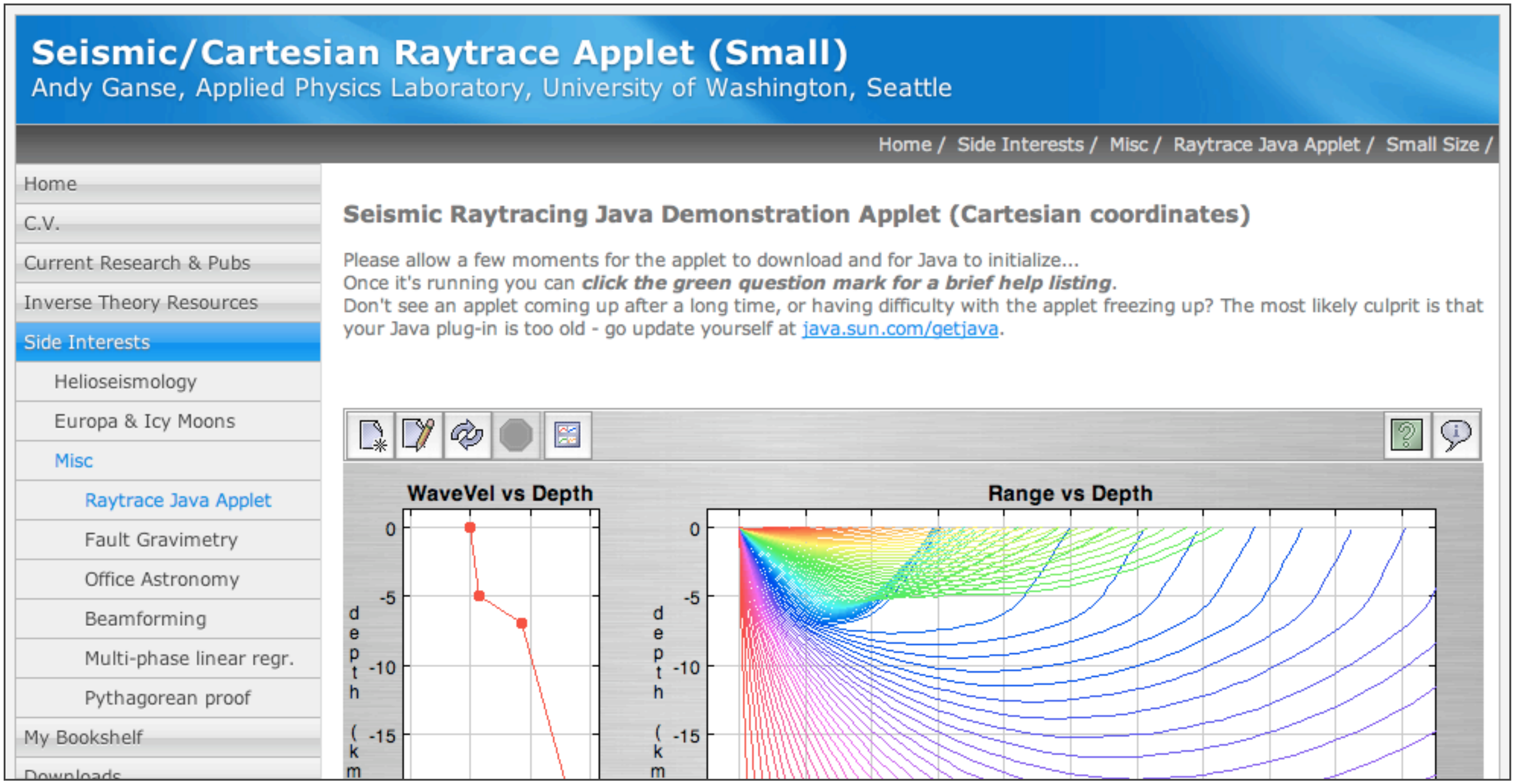
(also linked via ESS and APL directory pages)



Fortunately, not too many shameless plugs…

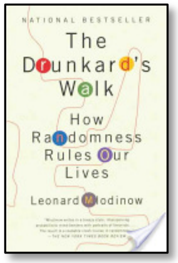
http://staff.washington.edu/aganse

(also linked via ESS and APL directory pages)

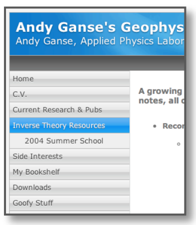


- Enter wave velocity profiles and watch the rays go!
- Spherical geometry one available too…

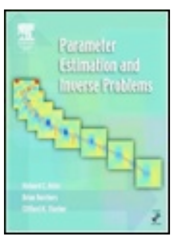
Recommended reading



Really fantastic popular book re probability and statistics:
The Drunkard's Walk,
by Leonard Mlodinow



My website (of course!) – pages on inverse theory resources, linear and nonlinear filter tutorial, ray-tracing, and much more.
http://staff.washington.edu/aganse



The best frequentist inverse theory textbook:
Parameter Estimation and Inverse Theory,
by Aster, Borchers, Thurber

The best Bayesian inverse theory textbook:
Inverse Problem Theory and Model Parameter Estimation,
by Albert Tarantola (available free online!)